# Evaluation of a Phasor-Based Fault Location Algorithm

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## INTRODUCTION

A relay by nature is a single ended device; that is, it only has access to the currents and voltages at one terminal of the protected line. With proper settings margins, the relay is able to use the data from a single line terminal to perform its protective functions. In order to protect the transmission line, the relay does not need an accurate estimate of the fault location; however, it is desirable to provide the most accurate distance to fault information possible to aid the user in locating the fault and taking corrective action to remove the cause of the fault. Many numeric relays available today incorporate some form of fault location estimation. Consider the one line diagram of Figure 1. The relay at terminal A sees the voltage $V_L$ and the current $I_L$. If breaker B is open, the voltage $V_L$ is a function of only $I_L$ (Equation 1). However, when breaker B is closed, the voltage $V_L$ is a function of both currents $I_L$ and $I_R$ (Equation 2).

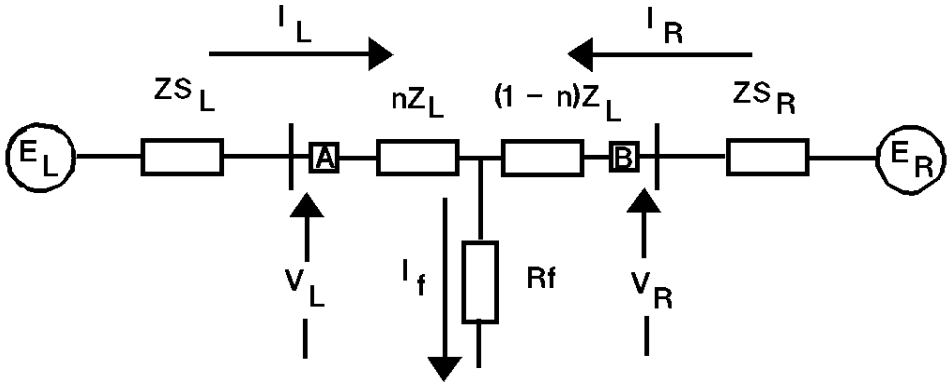


Figure 1 - Simplified System Diagram

$$V_L = I_L \bullet n \bullet Z_L + I_L \bullet Rf \quad (1)$$

$$V_L = I_L \bullet n \bullet Z_L + If \bullet Rf \quad (2)$$

Where:

$V_L$ is the relay voltage at terminal A.
$I_L$ is the relay current at terminal A.
$Z_L$ is the line impedance.
Rf is the resistance in the fault.
n is the per unit line length from terminal A to the fault.
If is the current flowing in the fault.

The effect of the infeed current from the remote terminal on the performance of the ground distance functions is well known [1]. It will, of course, also have an affect on the fault location calculation. Load flow over the line may also lead to errors in the fault location.

## FAULT LOCATION ALGORITHM

The earliest GE digital distance relays used the differential equation for protection. In this approach, the protection algorithm calculates the value of the inductance from the relay location to the point of fault. The inductance to the fault was then used for both distance protection and as an indication of the fault location. While this method worked well for some system and fault conditions, it became inaccurate when fault resistance was introduced.

The present GE digital distance relays use phasor quantities developed by discrete Fourier transforms rather than instantaneous samples for the protection algorithms. It was decided to use a fault location algorithm based on phasor values as well. An additional challenge was to reduce the effect of fault resistance and load flow on the calculation of fault location.

The " lumped parameter" approach of Equation 2 is used. The phasor quantities for the measured relay currents and voltages are computed by the Discrete Fourier Transform algorithm. Equation 2 is generalized and the quantities used will vary with the type of fault. Therefore this fault location algorithm will also require a separate algorithm to determine the fault type.

The fault current If is comprised of two components, one from each of the terminals, and may be expressed as:

$$If = If_L + If_R \quad (3)$$

Where:

$If_L$ is the fault current contribution from terminal A.
$If_R$ is the fault current contribution from terminal B.

Thus, Equation 2 may be rewritten as:

$$V_L = I_L \bullet n \bullet Z_L + (If_L + If_R) \bullet Rf \quad (4)$$

The currents $If_L$ and $If_R$ are not directly available to the relays. For this algorithm, the local current contribution to the total fault current is the total relay current during the fault minus the prefault load current. However, since the relay is using a single ended algorithm, each relay can only determine its own contribution to the fault.

Let the term $(If_L + If_R) \bullet Rf$ be represented by $If_L \bullet Rf_L$, where $Rf_L$ represents the apparent fault resistance seen by the relay at terminal A, and is equal to:

$$Rf_L = Rf \bullet \left( 1 + \frac{If_R}{If_L} \right) \tag{5}$$

$V_L$ may now be expressed by an equation that is in terms of only those quantities available to the relay at terminal A:

$$V_L = I_L \bullet n \bullet Z_L + If_L \bullet Rf_L \tag{6}$$

The term $Rf_L$ will be a real number (resistive component) provided that the currents $If_L$ and $If_R$ are in phase. In order for the currents to be in phase, the system shown in Figure 1 must be homogeneous; that is, the source and line impedances must all have the same impedance angle. While this is rarely the case in an actual power system, the assumption is made here to show how the fault location algorithm eliminates the effect of fault resistance. In practice, the typical non-homogeneous system will produce only a small error in the fault location estimate. Because the elements of Equation 6 are complex phasors, Equation 6 may now be separated into the real and imaginary components using the current and voltage values that are produced by the DFT algorithm.

$$V_LS + jV_LC = n \bullet [(I_L \bullet Z_L)S + j(I_L \bullet Z_L)C] + Rf_L \bullet (If_LS + jIf_LC) \tag{7}$$

Where:

$V_LS$ is the real component (sine term) of $V_L$.
$V_LC$ is the imaginary component (cosine term) of $V_L$.
$(I_L \bullet Z_L)S$ is the real component (sine term) of $(I_L \bullet Z_L)$.
$(I_L \bullet Z_L)C$ is the imaginary component (cosine term) of $(I_L \bullet Z_L)$.
$If_LS$ is the real component (sine term) of $If_L$.
$If_LC$ is the imaginary component (cosine term) of $If_L$.

Equating the real and imaginary components of equation 7 yields:

$$V_LS = n \bullet (I_L \bullet Z_L)S + Rf_L \bullet If_LS \tag{8}$$
$$V_LC = n \bullet (I_L \bullet Z_L)C + Rf_L \bullet If_LC \tag{9}$$

Equations 8 and 9 can now be solved to yield an equation for n, the per unit fault impedance to the fault. Multiply equation 8 by the term $If_LC$, and equation 9 by the term $If_LS$ to produce:

$$V_LS \bullet If_LC = n \bullet (I_L \bullet Z_L)S \bullet If_LC + Rf_L \bullet If_LS \bullet If_LC \tag{10}$$
$$V_LC \bullet If_LS = n \bullet (I_L \bullet Z_L)C \bullet If_LS + Rf_L \bullet If_LC \bullet If_LS \tag{11}$$

Subtracting equation 11 from equation 10 results in:

$$V_LS \bullet If_LC - V_LC \bullet If_LS = n \bullet [(I_L \bullet Z_L)S \bullet If_LC - (I_L \bullet Z_L)C \bullet If_LS] \quad (12)$$

Equation 12 may be solved for n.

$$n = \frac{V_LS \bullet If_LC - V_LC \bullet If_LS}{(I_L \bullet Z_L)S \bullet If_LC - (I_L \bullet Z_L)C \bullet If_LS} \quad (13)$$

Equation 13 shows that the fault location can be expressed in terms of the quantities available to the relay and that the fault resistance component, $Rf_L$, has been eliminated. Note that the above equations use the real and imaginary parts of the phasor quantity, $I_L \bullet Z_L$. This term reflects the fact that the current signal, $I_L$, has been processd by a digital mimic algorithm (digital transactor), and thus the dc offset component has been removed from the phasor, $I_L \bullet Z_L$.

As stated previously, the proper voltage and current phasors must be used to solve for n, and those phasors are determined by the fault type. The proper phasors are:

AG faults:

$V_L = V_A$

$I_L \bullet Z_L = ( I_A - I_0 ) \bullet ZR1 + K0 \bullet I_0 \bullet ZR0$

$If_L = ( I_A - I_{A\text{-}LOAD} )$

Where:

ZR1 is the positive sequence replica impedance of the line.

ZR0 is the zero sequence replica impedance of the line.

K0 is the ratio of the zero sequence impedance of the line to the positive sequence impedance of the line.

The calculations for BG and CG faults use the appropriate phase quantities.

AB faults:

$V_L = V_A - V_B$

$I_L \bullet Z_L = ( I_A - I_B ) \bullet ZR1$

$If_L = ( I_{AB} - I_{AB\text{-}LOAD} )$

The calculations for BC and CA faults use the appropriate phase to phase quantities.

Double line to ground faults are typically evaluated as phase to phase faults for fault location.

## STEADY STATE EVALUATION

The system of Figure 2 is used to provide a steady state analysis of the fault location algorithm and to demonstrate the effects of fault resistance and load on the accuracy of the algorithm. The

system of Figure 2 is homogeneous. The source to line impedance ratio at the left is 0.1, and at the right is 1. Phase A to ground faults were applied at 25 % and 75% of the line from the left terminal.

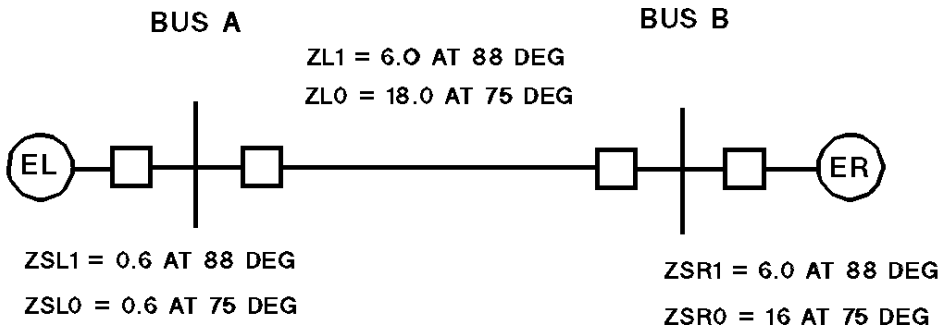


Figure 2 - Steady State Evaluation

The results of 24 fault cases are shown in Table I. Theses cases represent faults with no load, and with a load current of 2.75 amperes flowing from left to right, and for fault resistances of 0.0, 10, and 100 ohms. The L and R in the case number refers to the left and right terminal in Figure 2.

TABLE I

| CASE | FAULT RESISTANCE | LOAD | FAULT LOCATION | CALCULATED n | PERCENT ERROR |
|---|---|---|---|---|---|
| 1L | 0 | None | 0.25 | 0.2499 | -0.04 |
| 2L | 10 | None | 0.25 | 0.2500 | 0.00 |
| 3L | 100 | None | 0.25 | 0.02500 | 0.00 |
| 4R | 0 | None | 0.25 | 0.2500 | 0.00 |
| 5R | 10 | None | 0.25 | 0.2504 | 0.16 |
| 6R | 100 | None | 0.25 | 0.2496 | -0.16 |
| 7L | 0 | Yes | 0.25 | 0.2500 | 0.00 |
| 8L | 10 | Yes | 0.25 | 0.2500 | 0.00 |
| 9L | 100 | Yes | 0.25 | 0.2499 | -0.04 |
| 10R | 0 | Yes | 0.25 | 0.2501 | 0.04 |
| 11R | 10 | Yes | 0.25 | 0.2501 | 0.04 |
| 12R | 100 | Yes | 0.25 | 0.2498 | -0.08 |
| 13L | 0 | None | 0.75 | 0.7500 | 0.00 |
| 14L | 10 | None | 0.75 | 0.7498 | -0.03 |
| 15L | 100 | None | 0.75 | 0.7499 | -0.01 |
| 16R | 0 | None | 0.75 | 0.7499 | -0.01 |
| 17R | 10 | None | 0.75 | 0.7499 | 0.00 |
| 18R | 100 | None | 0.75 | 0.7455 | -0.59 |
| 19L | 0 | Yes | 0.75 | 0.7499 | -0.01 |
| 20L | 10 | Yes | 0.75 | 0.7500 | 0.00 |
| 21L | 100 | Yes | 0.75 | 0.7501 | 0.01 |
| 22R | 0 | Yes | 0.75 | 0.7501 | 0.01 |
| 23R | 10 | Yes | 0.75 | 0.7494 | -0.08 |
| 24R | 100 | Yes | 0.75 | 0.7479 | -0.28 |

The largest error for these cases is less than 0.6 percent. As can be seen from the estimates of the fault location, this algorithm is virtually unaffected by fault resistance and load current when applied on a homogeneous system.

## ERROR SOURCES

When a single ended fault location algorithm is actually used in a protective relay, the results will differ from the " ideal" shown in Table I. Inaccuracies in the fault location calculation may be caused by error sources in several areas.

### Power System

As mentioned previously, this particular algorithm is susceptible to errors caused by a non-homogeneous impedances in the power system. Its accuracy will also be affected by shunt capacitive current, by mutual coupling between parallel lines, as well as infeed current from a third terminal. The primary transducers, cts and ccvts, may introduce errors that can not be accounted for in the fault location calculation. All ground distance relays, and most fault locators, are based on the assumption that each of the phase wires in a transmission line has the same impedance. Depending upon the construction of the line, this may not be the case. The margins used in setting phase and ground distance relays more than compensate for this difference in the impedance on each phase. However, as more accurate fault locators are desired, the error introduced by the unequal impedances will become a more important factor.

### Relay Input Circuits

Typically the secondary currents and voltages applied to a relay may be isolated from the system and scaled to levels usable by the relay by a magnetic device such as a pt or ct. Input signals are processed by anti-aliasing filters and analog to digital converters (A/D). Each of these devices will add some error to the accuracy of the measurement. Some of these errors can be removed by calibration of the relay input circuits. Some uncertainty, however, will still remain. Appendix A includes a brief discussion of possible errors introduced into the measurements by the A/D conversion. The accuracy of the fault location estimate will be no better than the accuracy of the current and voltage measurements. There are methods available to improve the accuracy of the current and voltage measurements. However, some of these require averaging over a time period. This is acceptable for metering purposes, but the fault quantities will exist on the power system for a minimum time, typically on the order of several cycles and the fault duration may not be long enough to use the averaging methods.

### Model/Algorithm

A fault location algorithm is based on certain assumptions. This algorithm assumes that the system impedance angles are homogeneous, that the phase impedances are equal, that there is no effect from mutual coupling. It ignores any shunt currents, such as capacitive charging current.

The algorithm can be modified, or a different algorithm can be substituted, to improve the fault location calculation.

## EFFECT OF A NON-HOMOGENEOUS SYSTEM

In order to demonstrate the error introduced in this algorithm if the power system is non-homogeneous, the system of Figure 2 was modified such that the source impedance angles were 10 degrees less than the line impedance angles. This represents an extreme condition and is not a typical example of a non-homogeneous system. The modified system impedances are shown in Figure 3.

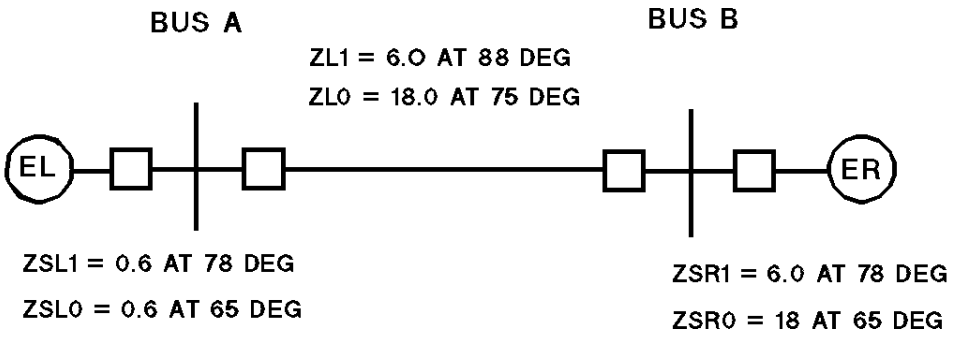


Figure 3 - Non-Homogeneous System

The results of the fault location calculation for the no load condition are shown in Table II. With load current, the errors will typically be larger than those shown in the Table. If the impedance angles of the source and line impedances are more closely matched then the error is reduced.

TABLE II

| CASE | FAULT RESISTANCE | LOAD | FAULT LOCATION | CALCULATED n | PERCENT ERROR |
|---|---|---|---|---|---|
| N1L | 0 | None | 0.25 | 0.2499 | -0.04 |
| N2L | 10 | None | 0.25 | 0.2612 | 4.5 |
| N3L | 100 | None | 0.25 | 0.3614 | 44.6 |
| N4R | 0 | None | 0.25 | 0.2500 | 0.00 |
| N5R | 10 | None | 0.25 | 0.2584 | 3.7 |
| N6R | 100 | None | 0.25 | -0.2628 | -205 |
| N7L | 0 | None | 0.75 | 0.7500 | 0.00 |
| N8L | 10 | None | 0.75 | 0.8325 | 11.0 |
| N9L | 100 | None | 0.75 | 1.5763 | 110 |
| N10R | 0 | None | 0.75 | 0.7501 | 0.01 |
| N11R | 10 | None | 0.75 | 0.8072 | 7.63 |
| N12R | 100 | None | 0.75 | 0.0800 | -89 |

The results shown in Table II indicate that the error in the fault location estimation increases as the fault resistance increases and as the actual fault location moves away from the relay location.

However, it can not be assumed that the " shorter" distance to fault estimate is the more accurate. Consider cases N3L and N12R. These cases represent the same AG fault at a distance of 25% of the line from the left terminal. The distance to fault estimate at the left is 0.36; from the right the estimate is only 0.08.

## SIMPLE TWO ENDED ALGORITHM

The fault location estimates calculated in Table II are the result of the single ended algorithm previously described in this paper. The use of this algorithm presupposed that the system was homogeneous; the large errors demonstrated in Table II are the result of the non-homogeneous nature of the system of Figure 3. The assumption of a homogeneous system allowed the currents $If_L$ and $If_R$ of Equation 4 to be at the same angle, and thus permitted Equation 4 to be expressed in terms of currents and voltages available to the relay at one terminal. Let us instead assume that the current from the remote terminal, $If_R$, is available for use in the fault location calculation. Equation 4 may then be used to derive the fault location estimate rather than the simplified expression of Equation 6. Table III shows the improvement in the fault location estimation for faults on the non-homogeneous system of Figure 3. The maximum error for these faults is less than 0.5%.

TABLE III

| CASE | FAULT RESISTANCE | LOAD | FAULT LOCATION | CALCULATED n | PERCENT ERROR |
|---|---|---|---|---|---|
| N1L | 0 | None | 0.25 | 0.2500 | 0.00 |
| N2L | 10 | None | 0.25 | 0.2501 | 0.04 |
| N3L | 100 | None | 0.25 | 0.2500 | 0.00 |
| N4R | 0 | None | 0.25 | 0.2500 | 0.00 |
| N5R | 10 | None | 0.25 | 0.2494 | -0.24 |
| N6R | 100 | None | 0.25 | 0.2496 | -0.16 |
| N7L | 0 | None | 0.75 | 0.7500 | 0.00 |
| N8L | 10 | None | 0.75 | 0.7498 | -0.03 |
| N9L | 100 | None | 0.75 | 0.7497 | -0.05 |
| N10R | 0 | None | 0.75 | 0.7501 | 0.01 |
| N11R | 10 | None | 0.75 | 0.7503 | 0.04 |
| N12R | 100 | None | 0.75 | 0.7499 | -0.01 |
| N13L | 0 | Yes | 0.25 | 0.2500 | 0.0 |
| N14L | 10 | Yes | 0.25 | 0.2499 | -0.04 |
| N15L | 100 | Yes | 0.25 | 0.2499 | -0.04 |
| N16R | 0 | Yes | 0.25 | 0.2501 | 0.04 |
| N17R | 10 | Yes | 0.25 | 0.2498 | -0.08 |
| N18R | 100 | Yes | 0.25 | 0.2489 | -0.44 |
| N19L | 0 | Yes | 0.75 | 0.7501 | 0.01 |
| N20L | 10 | Yes | 0.75 | 0.7501 | 0.01 |
| N21L | 100 | Yes | 0.75 | 0.7506 | 0.08 |
| N22R | 0 | Yes | 0.75 | 0.7481 | -0.25 |
| N23R | 10 | Yes | 0.75 | 0.7507 | 0.10 |
| N24R | 100 | Yes | 0.75 | 0.7505 | 0.07 |

The requirement to have the remote current available to the local relay raises several problems. If the relay itself is to perform the calculations, some means of data communications must exist between the relays. However, because the protection algorithms do not generally require an accurate fault location estimate, the current data from the remote terminal is not required in " real time" as it is in a current differential scheme. In fact, the double ended fault location calculation does not need to be performed in the relay at all. Data from both terminals may be transmitted to a PC either manually or via substation integration for post disturbance fault location calculation.

Because the sampled current and voltage data is obtained from independent devices, some method of " synchronizing" the data must be used. This subject has been discussed in previous papers. Exact synchronizing of the samples may be required when the instantaneous values are used in the fault location algorithm. Since this particular algorithm employs phasor quantities rather than instantaneous samples, exact synchronizing of the samples is not required when combining data from different ends of a transmission line. The phasors from each line terminal must share a common reference to establish the correct phase angles. The prefault currents and voltages may be used to estimate the proper phase angle for the required remote current.

## CONCLUSION

Fault location calculations based on quantities from a single line terminal are subject to errors that can not be completely eliminated by settings in the algorithm. The effect of many of the error sources can be reduced to the extent that the single ended algorithm can produce a reasonable first approximation of the actual fault location. A much better estimate of the fault location can be obtained when currents from other lines and/or from other terminals are used in the calculation. These multi-ended fault location calculations could be done in the digital relay if the remote end currents are made available to the local relay via some communications path, or they can be easily done off-line in a PC based program.

## REFERENCES


[1] G. E. Alexander and J. G. Andrichak, " Ground Distance Relaying: Problems and Principles", Eighteenth Annual Western Protective Relaying Conference, October 1991.

[2] DLP- Digital Line Protection, GE technical publication GET-8037.

[3] A. G. Phadke and J. S. Thorp, Computer Relaying for Power Systems, New York: John Wiley and Sons Inc., 1990.

[4] J. M. Kennedy and G. E. Alexander, " Variable Digital Filter Response Time in a Digital Distance Relay", Twentieth Annual Western Protective Relaying Conference, October 1993.

[5] W. A. Elmore, " Zero Sequence Mutual Effects on Ground Distance Relays and Fault Locators", Nineteenth Annual Western Protective Relaying Conference, October 1992.

[6] D. J. Lawence, L. Z. Cabeza, and L. T. Hochberg, " Development of an Advanced Transmission Line Fault Location System: Part II - Algorithm Development and Simulation", IEEE PES Summer Meeting, July 1990.

[7] D. Novosel, D. G. Hart, E. Udren, and J. Garitty, " Unsynchronized Two-Terminal Fault Location Estimation", IEEE PES Winter Meeting, January, 1995.

## APPENDIX A

### Sample A/D Resolution Calculation

The calculation of the accuracy of the analog to digital conversion has been described fully elsewhere [3]. This is simply intended as a demonstration of how the current range required for protection may adversely affect the accuracy of the measured currents, and thus the accuracy of the fault location calculation.

Assume that the relay has a 12 bit A/D, has a nominal current rating of 5 amperes, and the requirement that the current measurements remain linear to 20 times rated. This relay must be capable of measuring an instantaneous current equal to the peak of a fully offset current with an rms value of 100 amperes, or 283 amperes. Based on this information, a single count, or bit, of data is equal to $283/(2^{11})$ or 0.138 amperes. This can be expressed as a percent of peak current for various levels of input as shown in Table A.

TABLE A

| Multiples of Rated | Peak Current | 1 bit as a percent of peak |
|---|---|---|
| 20 (fully offset) | 283 | 0.049 % |
| 20 (no offset) | 141.5 | 0.098 % |
| 1 (no offset) | 7.07 | 1.95 % |
| 0.1 (no offset) | 0.707 | 19.5 % |

The errors introduced by the A/D resolution are based on the maximum current for which the relay was designed. As a result, the errors are larger as the input current is reduced.

Due to the fault resistance, several of the currents calculated for the faults used in this paper were less than 0.1 per unit amperes. For the purposes of this paper, any errors introduced by the A/D process were ignored.

There are methods available to reduce the errors introduced by the large dynamic range required for protective relays, such as multiple A/D ranges; but for any given A/D range, the error will be larger as the current is reduced. It is also possible to reduce the effect of the A/D errors by other means such as average over a time period. However, the fault location estimate must be made on currents which last, hopefully, for no more than a few cycles and which may contain non-fundamental components such as dc offset in the current and ccvt transients in the voltage.